Idealized Numerical Modeling Experiments of the Diurnal Cycle of Tropical Cyclones

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Numerical experiments are performed to evaluate the role of the daily cycle of radiation on axisymmetric hurricane structure. Although a diurnal response in the high cloudiness of tropical cyclones (TCs) has been well documented in the past, the impact to storm structure and intensity remains unknown. Previous modeling work attributes differences in results to experimental setup (e.g., initial and boundary conditions) as well as to radiative parameterization schemes. Here, a numerically-simulated TC in a statistical steady-state is examined to quantify the TC response to the daily cycle of radiation, and a modified, Sawyer–Eliassen approach is applied to evaluate the dynamical mechanism.

Fourier analysis in time reveals a spatially coherent pattern in the temperature, wind, and latent heating tendency fields that is statistically significant at the 95% level. This signal accounts for up to 62% of the variance in the temperature field of the upper troposphere, and is mainly concentrated in the TC outflow layer. Composite analysis reveals a cycle in the storm intensity in the low-levels, which lags a periodic response in the latent heating tendency by 6 h. Average magnitudes of the azimuthal wind anomalies near the radius of maximum wind (RMW) are 1 m s$^{-1}$, and account for 21% of the overall variance. A hypothesis is drawn from these results that the TC diurnal cycle is comprised of two distinct, periodic circulations: (1) a radiatively-driven circulation in the TC outflow layer due to absorption of solar radiation, and (2) a convectively-driven circulation in the lower and middle troposphere.
due to anomalous latent heating from convection. These responses are coupled and are periodic with respect to the diurnal cycle.

Using a modified, Sawyer-Eliassen approach for time-varying heating, these hypotheses are evaluated to determine the impact of periodic diurnal heating on a balanced vortex. Periodic heating near the top of the vortex produces a local overturning circulation in the region of heating that manifests as inertia–buoyancy waves in the storm environment. Periodic heating in the lower troposphere drives an overturning circulation that resembles the Sawyer–Eliassen solution. This low-level heating induces a positive perturbation azimuthal wind response of 4 m s$^{-1}$ near the RMW, which lags the maximum in streamfunction by 6 h. Comparison of these solutions to the numerically-simulated TC reveals a close correspondence of results, suggesting that the axisymmetric TC diurnal cycle is a balanced response driven by periodic heating.

The sensitivity of these results to the length of the diurnal period and the vortex intensity are evaluated using the modified, Sawyer–Eliassen approach. Although the true diurnal period is fixed in nature, these experiments allow for the relationship between the magnitude and structure of the TC diurnal signal to the length of the diurnal period to be explored. Results demonstrate that the TC diurnal cycle exhibits large variance, even for the same heating distributions. High-frequency forcing projects mainly onto inertia–buoyancy waves, while low-frequency produces a balanced response resembling the Sawyer–Eliassen solution. Comparison to two, numerically simulated TCs with modified diurnal periods show similar results. In addition, stronger diurnal signals are observed for stronger vortices, suggesting a dependence of the TC diurnal signal on the underlying state of the vortex. These results imply that the magnitude and structure of the TC diurnal signal in nature varies throughout the storm lifetime, and is a function of the structure and intensity of the vortex.
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DEDICATION

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Chapter 1

INTRODUCTION

The diurnal cycle of radiation is a fundamental component of the climate system that is associated with well-defined variations in solar forcing. Convection in the tropics exhibits a clear diurnal signal, with continental convection demonstrating a maximum in the late afternoon and early evening, and oceanic deep convection demonstrating a maximum in the early morning (Gray and Jacobson, 1977, Yang and Slingo, 2001). Over the ocean, contributions to the diurnal signal include convection in the intertropical convergence zone, island and coastal convection that propagates offshore, tropical warm pool convection, and mesoscale convective systems (MCSs) that form over Africa (Houze et al., 1981, Mapes and Houze, 1993, Bain et al., 2010). Early morning maxima in precipitation are also observed over the ocean (Nesbitt and Zipser, 2003). Similarly, a coherent diurnal signal in the high clouds of tropical cyclones (TCs) has been well documented in the past; however, the dynamical mechanism and the impact to storm structure and intensity are essentially unknown. Although this signal has been previously linked to storm intensity, the reason is poorly understood (Browner et al., 1977).

1.1 Background: The Diurnal Cycle of Tropical Cyclones

Previous work on the diurnal cycle of TCs has focused mainly on documenting storm high-cloudiness. For convection in TCs, Browner et al. (1977) showed that in the Atlantic, the maximum area coverage of high clouds in storms occurs at 1700 local time, with a minimum at 0300. In the Pacific, Muramatsu (1983) finds a maximum in the early morning (near 0600-0730 LT) and a minimum in the evening (around 1800-2100 LT). For TCs near Australia, Lajoie and Butterworth (1984) demonstrate a maximum at 0300 LT and a minimum at
1800 LT. Steranka et al. (1984) clarify the inconsistency in timing between ocean basins by showing that these calculations are sensitive to the chosen brightness temperature, as well as to the radial distance from storm center. By organizing storms into sets of concentric rings, Steranka et al. (1984) demonstrate that the diurnal cloud maximum is a propagating feature that emanates from the storm center. Estimated propagation speeds are $10-15 \text{ m s}^{-1}$ for tropical storms and $2 \text{ m s}^{-1}$ for hurricanes.

Recent observational studies have focused on the upper-level clouds, and further document a diurnal, as well as a semi-diurnal, signal in the TC cirrus canopy. Using Hovmöller diagrams of infrared brightness temperature, Kossin (2002) demonstrates a pronounced diurnal oscillation in the areal extent of the TC cirrus canopy. Power spectrum analysis reveals that the diurnal signal is often absent near the convection in the inner core of the storm, but a significant semidiurnal signal is occasionally observed in this region. Kossin (2002) shows that the semidiurnal signal is in phase with the semidiurnal atmospheric tide ($S_2$), and hypothesizes that this relationship results in an oscillation of the lapse rates, which modulates the convection. Similar to Steranka et al. (1984), Dunion et al. (2014) also document an outward propagating TC diurnal signal. Using 6-hour differences in brightness temperatures, Dunion et al. (2014) show that for North Atlantic major hurricanes from 2001 to 2010 diurnal “pulses” originate near the storm center around sunset, strengthen overnight, and propagate outward in the early morning with propagation speeds near 5-10 m s$^{-1}$. Significant warming of the cloud tops is observed after the passing of the diurnal pulse, which is coincident with an observed radial expansion of storm’s overall structure.

The TC diurnal cycle has also been explored using numerical experiments, with varying results. Sundqvist (1970) and Hobgood (1986) demonstrate a more rapid growth rate in the developing stages of the simulation of a single storm with a diurnal cycle, with no effect thereafter. Hack (1980) and Tuleya and Kurihara (1981) find no influence on the initial growth rate, but rather an earlier onset of storm intensification. For axisymmetric TCs, Craig (1996) demonstrates that a modest increase in overall storm intensity is observed with realistic radiation, but shows no impact on the storm growth rate or the timing of intensifica-
tion. Using modified radiation experiments in the Advanced Research Weather Research and Forecasting model (WRF-ARW), Melhauser and Zhang (2014) show that TC development is highly sensitive to the diurnal cycle, exhibiting suppressed formation in daytime-only and no-radiation experiments and accelerated intensification in nighttime-only experiments as compared with a control. Tang and Zhang (2016) demonstrate a similar result for Hurricane Edouard (2014), showing that the impact of the diurnal cycle is mainly concentrated on the rate of intensification for the development stage of the TC, and on the structure and strength in the rapid intensification and mature stage. In the mature stage, Tang and Zhang (2016) note that the difference in radiation between the daytime and nighttime phase has a minor impact on storm intensity.

Presently, there is no consensus on the impact of the diurnal cycle of radiation on TC structure and intensity. The current hypothesized mechanisms can be summarized as follows: (1) radiative destabilization at cloud top overnight, which steepens the lapse rate and causes an increase in nocturnal convection (Hobgood, 1986); (2) differential cooling between the cloudy and clear air environment of the TC, which enhances low-level convergence (Gray and Jacobson, 1977, Melhauser and Zhang, 2014, Tang and Zhang, 2016); (3) radial contraction of the upper branch of the TC secondary circulation by outgoing longwave radiation, shrinking the extent of the cirrus canopy overnight (Kossin, 2002); While this disagreement motivates the need for a dynamical explanation of this phenomenon, the direct evaluation of these hypotheses is not the objective of the current study.

1.2 Goal and Dissertation Outline

Here, a numerical experiment using a long, statistically steady–state simulation of an axisymmetric TC is performed to evaluate the role of the diurnal cycle of radiation on storm structure and intensity. The goal of this dissertation is to quantitatively describe the TC diurnal cycle and investigate the dynamical mechanism in a numerical simulation. A steady-state TC is produced with no environmental influences to isolate the internal storm response to the diurnal cycle of radiation. Previous studies have used this framework to explore the
impact of radiation on TC variability (Hakim, 2011, 2013, Brown and Hakim, 2013); however, they did not include a diurnal cycle. Based on the lack of conceptual understanding of this phenomenon, we first evaluate the TC diurnal cycle using an axisymmetric model. Then, following the work of Willoughby (2009), a modified, Sawyer–Eliassen approach is applied to investigate the role of periodic diurnal heating on a balanced vortex. Hypotheses drawn from the CM1 simulation are tested, and the sensitivity of these results to the period of forcing as well as the vortex intensity is diagnosed.

This thesis is organized as follows: Chapter 2 describes the numerical simulation and the compositing method, and provides the statistical analysis of the TC diurnal cycle. A dynamical connection between the TC diurnal cycle in the wind field and periodic heating is proposed. Chapter 3 uses the modified, Sawyer–Eliassen approach of Willoughby (2009) to evaluate the periodic heating hypothesis, and compares the results to the CM1 solutions. Chapter 4 examines the sensitivity of the these solutions to the period of forcing as well as the mean vortex intensity. Chapter 5 provides a summary and conclusions.
Chapter 2

IDEALIZED NUMERICAL EXPERIMENTS OF THE DIURNAL CYCLE OF TROPICAL CYCLONES

The model configuration and the methods for analysis are introduced first, followed by the statistical analysis and documentation of the CM1 TC diurnal cycle.

Sections from the chapter are included in the following publication:


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2.1 Method

2.1.1 Model Configuration and Analysis

The model used here is the axisymmetric, non-hydrostatic cloud model of Bryan and Fritsch (2002) and Bryan and Rotunno (2009b) (CM1). This model includes the National Aeronautics and Space Administration (NASA) Goddard radiation scheme (Chou and Suarez, 1994), simulating the effects of both longwave and shortwave radiation, and uses the NASA-Goddard version of the Lin et al. (1983) ice microphysics scheme. The domain is 1500 km in the horizontal direction and 25 km in the vertical direction. A fixed horizontal resolution of 4 km is used; resolution in the vertical is 250 m from 0 to 10 km, with a gradual increase to 1 km at the top. A damping layer is applied to momentum and potential temperature within 5 km of the model top, and to momentum within 100 km of the outermost radius. The horizontal and vertical length scales for the turbulence parameterization are 500 and 200 m, respectively, and are consistent with both previous axisymmetric modeling studies in
CM1 (Bryan and Rotunno, 2009a, Rotunno and Bryan, 2012, Hakim, 2011) as well as recent work measuring the horizontal mixing length in the low–level region of intense hurricanes (Zhang and Montgomery, 2012). The exchange coefficients for the surface fluxes of energy and momentum are given a ratio of unity, and initial temperature and moisture profiles are taken from Rotunno and Emanuel (1987).

Following the approach established by Hakim (2011, 2013) and Brown and Hakim (2013), we integrate the simulation to 340 days, of which approximately 40 days are required for the storm to reach radiative-convective equilibrium with the environment. This simulation excludes all external influences, such as varying sea surface temperature (SST) and wind shear, which allows for an identification of storm structure and intensity with the diurnal cycle of radiation. The sea surface temperature is fixed at 26.3 degrees C, and the Coriolis parameter is constant at a value corresponding to 20 degrees latitude. The solution is obtained for a fixed calendar day of September 10, 2015. Since angular momentum is lost in the radial inflow layer due to frictional dissipation, a source of angular momentum is required in the closed system to maintain statistically–steady state. This is accomplished at the lateral boundary, where anticyclonic (i.e., negative v component) parcels in the outflow reach the damping layer near the lateral edge of the domain, which increases their angular momentum. The damping coefficient is fixed in time and therefore does not vary periodically with respect to the diurnal cycle.

Using the last 300 days of the simulation, power spectrum analysis is performed to identify the diurnal harmonics, and time-series reconstruction is used to quantify the diurnal cycle. Composite analysis is then used to evaluate the horizontal and vertical structure of the TC diurnal cycle. To compute the composites, the time-mean over the 300-day sample is first removed; then, the anomalies from this mean are composited at each hour of the day (e.g., all the azimuthal wind anomalies at 1:00a.m., 2:00a.m., etc.).

\[1\) In real storms, this angular momentum source is supplied by lateral mixing with the storm environment.
2.1.2 The Equilibrium Storm

Fig. 2.1a shows the time-series of the maximum surface wind for the full 340-day simulation. After a transient spin-up period lasting approximately 40 days, the storm reaches radiative-convective equilibrium about a statistically–steady state. The steady–state portion of the TC simulation will be referred to as the “equilibrium storm”. The mean intensity at the surface for the full simulation is 36 m s$^{-1}$, with a standard deviation of 3.9 m s$^{-1}$. Peak values of maximum surface wind speeds exceed 50 m s$^{-1}$. The location of the radius of maximum wind also exhibits large variability (Fig. 2.1b); this radius quickly transitions from the time-mean value of 53 km to values near 100–120 km approximately 40 times, indicating possible eyewall replacement cycles. Properties of the transient, super–intense storm and eyewall replacement cycles are discussed in detail in Hakim (2011, 2013); here we will only discuss properties related to the diurnal cycle.

The time-mean values of the equilibrium storm for the azimuthal wind, temperature, radial wind, and latent heating tendency fields are plotted in Fig. 2.2a-d. These fields serve as the baseline for comparison with the composite anomalies presented in this study. A maximum in azimuthal wind is observed at a height of 1.1 km and a radius of 50 km (Fig. 2.2a), demonstrating the mean location of the radius of maximum wind for the equilibrium storm. The average magnitude at this level is 38 m s$^{-1}$. Winds decay rapidly with both radius and height, and become negative at upper levels at radii greater than 300 km. Absolute temperatures in the troposphere decrease nearly linearly with height (Fig. 2.2b); a mean lapse rate of -6.1$^\circ$K km$^{-1}$ is observed from the surface up to 12.5 km. The height of the tropopause is indicated by the abrupt transition in the gradient of absolute temperature near 13 km. In the lower stratosphere, absolute temperature is more homogeneous with a lapse rate of 0.3$^\circ$K km$^{-1}$. Negative values of radial wind are observed near the surface, indicating radial inflow (Fig. 2.2c). Magnitudes approach -10 m s$^{-1}$ near the storm core and extend from the radius of maximum wind to approximately 400 km. Positive values, indicating radial outflow, are observed at a height 12.5 km with an average magnitude of
4 m s\(^{-1}\). In the outer environment, descending mid-level radial inflow is observed near a height of 5 km and from radii of 225–400 km. The average magnitude of this inflow is -2 m s\(^{-1}\). A maximum in the latent heating tendency is observed in the troposphere (Fig. 2.2d); tendencies are highest at the radius of maximum wind with magnitudes near 11 K h\(^{-1}\). These values decay rapidly with increasing radius and height, reaching zero at 75 km and 8 km, respectively. Negative latent heating tendency is observed at a radius of 100 km and 4 km height, and is due to the melting of ice particles near the freezing level (not shown). Negative latent heating tendency in the boundary layer demonstrates evaporation of liquid water into the subsaturated air, as well as evaporative cooling from precipitation.
Figure 2.2: The time-mean (a) azimuthal wind (m s\(^{-1}\)), (b) temperature (K), (c) radial wind (m s\(^{-1}\)) and (d) latent heating tendency (K h\(^{-1}\)) for the equilibrium storm. Negative values in (c) represent radial inflow.

### 2.2 Results

Results from time-series analysis of the experiment are presented first, followed by results based on composite analysis of the diurnal cycle.

#### 2.2.1 Time-series Analysis

Fig. 2.3a shows the time series in the azimuthal wind field for the equilibrium storm at a radius of 50 km and a height of 1.1 km, which corresponds to the mean location of the radius of maximum wind. The time mean has been removed, and a high-pass filter has been applied to remove low frequencies corresponding to periods greater than 5 days, as this variability is not a subject of this study. The high-pass filtered time series has a standard deviation of
Figure 2.3: The azimuthal wind time series (m s\(^{-1}\)) for (a) the equilibrium storm and (b) days 10 through 30 at twice the mean location of the radius of maximum azimuthal velocity. The time-mean has been removed, and the data has been high-pass filtered to remove low frequency variability with periods greater than 5 days. Dashed lines indicate the addition and subtraction of three standard deviations from the mean value.

2.3 m s\(^{-1}\) at this location, with peak amplitudes of the wind at individual times exceeding 7 m s\(^{-1}\). Magnifying a 30-day portion of the high-pass filtered time series shows that multiple high frequencies are present (Fig. 2.3b); however, examining the time series on days 19, 21, and 22, for example, suggest an underlying diurnal signal.

Fig. 2.4 shows the power spectrum of the high-pass filtered time series in the azimuthal wind at a radius of 102 km and height of 1.1 km, which is approximately double the distance of the radius of maximum wind. The peak of the first diurnal harmonic, corresponding to a period of 24 h, is the dominant peak of the spectrum. Statistical significance of the peaks is determined by comparing the power spectrum to a null hypothesis defined by an
Figure 2.4: Power spectrum of the azimuthal wind (m² s⁻²) at twice the mean location of the radius of maximum wind (solid black line). The red-noise fit (red line) and the 95% confidence bound (dashed blue line) are included for reference. Data has been high-pass filtered to remove low frequency variability with periods greater than 5 days.

AR-1 process, based on the 1-hour autocorrelation. Frequencies where the power exceeds the 95% confidence bound on the AR-1 process, defined by an chi–squared test (Wilks, 2005), are potentially significant. The first diurnal harmonic clearly meets this requirement. Peaks corresponding to periods of 48 h, 20 h, 12 h, and 8.5 h also meet this requirement; however, these peaks are within the expected uncertainty range for the 95% confidence bound threshold. All remaining frequencies are considered to be consistent with the red-noise null hypothesis.

Calculating the power spectrum and applying the null hypothesis to each grid point on the domain demonstrates the spatial distribution of the first diurnal harmonic in the

---

²The present study only considers the diurnal signal (i.e, the first diurnal harmonic). Further investigation of the potential significance of these peaks is beyond the scope of this work.
azimuthal wind field (Fig. 2.5a). Two main regions are highlighted: the first region is in the upper troposphere at a radius of 58 km and a height of 13.5 km, and at radii between 100 km and 300 km; the second region is near the surface at a radius of 75 km and up to a height of 5 km, and from 150 km in radius extending to 400 km. Weak significance is also indicated at radii between 100–200 km and at 11.5 km height and from radii between 150–400 km and at 19.5 km height. Everywhere else on the domain the diurnal cycle is not statistically significant. Significant power in the absolute temperature (Fig. 2.5b) and the radial wind (Fig. 2.5c) fields also highlights the same two main regions, with maxima in the upper troposphere. A third region of importance in the mid–troposphere is indicated by the latent heating tendency (Fig. 2.5d); this region extends from a height of 1.1 km to a height of 8 km, and is narrow, spanning the eyewall region from approximately 40 km to 90 km. Power in these highlighted regions suggests two things: (1) a coherent diurnal signal is detected at the top of the storm and (2) the diurnal cycle exhibits an interior storm response. The low-level signal that extends from the inner core of the storm ($r < 150$ km) to the outer environment ($r > 300$ km) in the azimuthal wind, temperature, and radial wind fields suggests the outer TC environment is also responding to the diurnal cycle of radiation.

Table 1 summarizes the fraction of the variance explained by the diurnal cycle for the azimuthal wind, temperature, radial wind, and latent heating tendency fields. A substantial amount of variance is explained in the TC outflow layer, where the diurnal cycle accounts for 62% of the overall variance in the temperature and 37% of the variance in the radial wind field. For the azimuthal wind, the diurnal cycle accounts for 36% of the variance in this region. In the boundary layer, the diurnal signal accounts for 28% of the variance in the temperature and 25% in the radial wind fields. The mid-level response is limited to the latent heating tendency, which explains 3% of the variance. These results suggest that the diurnal cycle contributes most significantly to variability of the TC outflow layer, with impact to the lower levels and the region outside the eyewall.
Figure 2.5: The absolute power in the first diurnal harmonic at each grid point location for the (a) azimuthal wind (m$^2$/s$^2$), (b) temperature (K$^2$), (c) radial wind (m$^2$/s$^2$) and (d) latent heating tendency (K$^2$/h$^2$). Colors represent only those regions with statistical significance at the 95% level.

2.2.2 Composite Analysis

Anomalies from the time-mean equilibrium storm are composited at each hour to determine the evolving structure of the fields related to the diurnal cycle. The radial structure of the composite azimuthal wind field reveals a symmetric, horizontally broad signal near the surface (Fig 2.6a). This field is a vertical average over the boundary layer, from the surface to 2 km height, and corresponds to the highlighted region at low levels previously discussed in (Fig 2.5a). Starting at 03:00 LT, a positive azimuthal wind anomaly is observed in the region near the eyewall. This positive anomaly has a maximum magnitude of 0.8 m s$^{-1}$ at 11:00 LT and is apparent mainly over 50–100 km radius. A weak negative anomaly is observed inside of the radius of maximum wind near 40 km, with average magnitude of 0.3 m s$^{-1}$. Beyond
Table 2.1: The variance explained by the diurnal cycle for the temperature, wind, and latent heating tendency fields for specific vertical levels in the storm. Data has been high-pass filtered to remove low frequency variability. Percentages represent the maximum amount of variance explained in this region.

<table>
<thead>
<tr>
<th></th>
<th>Outflow Layer</th>
<th>Mid-troposphere</th>
<th>Boundary Layer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Azimuthal Wind</td>
<td>36%</td>
<td>-</td>
<td>21%</td>
</tr>
<tr>
<td>Temperature</td>
<td>62%</td>
<td>-</td>
<td>28%</td>
</tr>
<tr>
<td>Radial Wind</td>
<td>37%</td>
<td>-</td>
<td>25%</td>
</tr>
<tr>
<td>Latent Heating Tendency</td>
<td>-</td>
<td>3%</td>
<td>-</td>
</tr>
</tbody>
</table>

100 km the positive signal weakens, with an average magnitude of 0.25 m s$^{-1}$. This positive azimuthal wind anomaly persists mainly until 15:00 LT, except for radii inwards of 50 km where a positive anomaly remains until 21:00 LT. A negative azimuthal wind anomaly forms at radii of 50–350 km at 15:00 LT, with a maximum value of 0.87 m s$^{-1}$ observed at 21:00 LT near the radius of maximum wind at 54 km. These signals are periodic, and suggest a diurnal response in intensity is occurring at the radius of maximum wind. The broad, weak signal at large radii indicates a diurnal, nearly stationary, “pulsing” of the intensity of the storm in the far field, which implies an effective change in storm size.

The radial structure of the latent heating tendency suggests a diurnal cycle of anomalous convection (Fig. 2.6b). This field is a vertical average from the surface to 10 km. A positive anomaly is observed beginning at 23:00 LT from 50–125 km in radius, which persists until the early morning. Maximum (minimum) values in latent heating tendency anomalies are 0.4 K h$^{-1}$. Positive latent heating tendency anomalies weaken substantially after 09:00 LT, which coincides with the onset of solar heating. Negative anomalies then form and persist throughout the day. In real storms, diurnal “pulses” in the high clouds form in the inner core near sunset, propagate away from the storm center overnight, and reach radii near 300-400 km in the outer environment the following afternoon (Steranka et al., 1984, Dunion et al., 2014). Significant warming near cloud top is observed behind the observed propagating feature. These numerical results are consistent with the timing and location of the diurnal pulses.
Figure 2.6: Hovmöller diagram for the (a) azimuthal wind (m s\(^{-1}\)) and (b) latent heating tendency (K h\(^{-1}\)) for vertical averages taken from the surface to 2 km height and from the surface 10 km height, respectively. Fields are composite anomalies computed at each hour.

from Steranka et al. (1984) and Dunion et al. (2014), suggesting that convection is favored overnight, with local maxima occurring near the eyewall region, and inhibited throughout the day. These results also suggest that convection near the surface is responding to the diurnal cycle, and is not only present in the high clouds.

The mean vertical structure of the TC diurnal cycle is depicted using the composite temperature field (Fig. 2.7a). A wave-like pattern is observed in both the lower troposphere as well as the stratosphere\(^3\), connected to anomalies that arise in the TC outflow layer. At a height of 12 km, positive temperature anomalies appear at 11:00 LT and last until 20:00 LT, which is consistent with warming by absorption of radiation. These anomalies slowly propagate downwards through the troposphere and reach a maximum at the top

\(^3\)The mean height of the tropopause in this simulation is 14.5 km.
of the boundary layer near 2 km approximately 6 hours later. Beginning at 22:00 LT, negative temperature anomalies are observed at a height of 12.5 km, consistent with cooling due to longwave emission from cloud top. Again, downward propagation of the signal is observed. Propagating disturbances are also observed in the stratosphere; beginning at 05:00 LT, negative temperature anomalies propagate from a height of 18 km down to the TC outflow layer at 12.5 km height throughout the day. As will be discussed below, such phase propagation is consistent with upward energy propagation for inertia-gravity waves.

The dispersion relationship for two-dimensional \((x-z)\), hydrostatic, Boussinesq gravity waves in a rotating, stably-stratified atmosphere at rest is given by

\[
\nu^2 = f^2 + \frac{N^2 k^2}{m^2}
\]  

(2.1)
where \( \nu \) is the frequency, \( f \) is the Coriolis parameter, \( N \) is the Brunt-Väisälä frequency, and \( k \) and \( m \) are the zonal and vertical wavenumbers, respectively. Note that the meridional wavenumber \( l \) is set to zero. For a TC, rapid rotation in the core of the storm requires consideration of the centrifugal force; however, far from the storm in the surrounding environment, the ratio of the centrifugal force to the Coriolis force is small, and the centrifugal term can be neglected. In this region, we can locally apply this dispersion relationship in a Cartesian framework to qualitatively estimate inertia–gravity wave features. Rearranging Eq. 2.1 for \( k \) and recalling that

\[
k = \frac{2\pi}{\lambda_x}, \quad m = \frac{2\pi}{\lambda_z}
\]

(2.2)

where \( \lambda_x \) and \( \lambda_z \) are the horizontal and vertical wavelengths, respectively, yields

\[
\lambda_x = \frac{N\lambda_z}{\sqrt{(2\pi/\tau)^2 - f^2}}
\]

(2.3)

where \( \nu = \frac{2\pi}{\tau} \) and \( \tau \) is the wave period. Estimating a propagation speed of 0.06 m s\(^{-1}\) from the downward propagating negative signal in the stratosphere of Fig. 2.7 and using the phase speed relationship \( c_{pz} = \lambda_z/\tau \) yields an estimated vertical wavelength of 5.6 km. Plugging in this value for \( \lambda_z \) in Eq. 2.3 and taking a value of \( 4 \times 10^{-4} \) s\(^{-2}\) for \( N^2 \) in the stratosphere, \( \tau = 24 \) h, and calculating \( f \) at a latitude of 20 degrees yields a horizontal wavelength of 2,124 km and a phase speed \( c_{px} = \lambda_x/\tau \) of 24.6 m s\(^{-1}\). For the troposphere, taking a value of \( 1 \times 10^{-4} \) s\(^{-2}\) for \( N^2 \) yields a horizontal wavelength of 1,062 km and a phase speed of 12.3 m s\(^{-1}\). These results are similar to estimates of diurnal propagating features in the high clouds of real storms, which have propagation speeds of 5-15 m s\(^{-1}\) (Steranka et al., 1984, Dunion et al., 2014), and suggest the response in the lower stratosphere is twice as fast as the response in the lower troposphere.

Examining the radial wind field also shows downward propagating features (Fig. 2.7b); these disturbances are observed from the lower stratosphere near 20 km height down to
10 km. Anomalous radial inflow is observed at the base of the TC outflow layer near 10 km from 11:00 LT until 22:00 LT with magnitude of 0.8 m s$^{-1}$. Anomalous radial outflow is observed at the top of the outflow layer near 13.5 km, with magnitude near 1 m s$^{-1}$. Overnight, the signal changes, with anomalous radial inflow observed above the TC outflow layer and anomalous radial outflow observed below 12.5 km, with magnitudes of -1 m s$^{-1}$ and 0.8 m s$^{-1}$, respectively. Durran et al. (2009) have shown using numerical experiments that heating of cirrus clouds in the tropical tropopause layer can produce vertically propagating disturbances; these perturbations extend well beyond the region of heating due to gravity wave dynamics. Durran et al. (2009) also demonstrate that one characteristic of cirrus-generated gravity waves is to drives radial inflow at the base of the heat source and radial outflow at the top, similar to the features described here. At the surface, anomalous radial outflow is observed from 14:00 LT to 20:00 LT, with magnitudes near 0.6 m s$^{-1}$. Radial inflow of similar magnitude is then observed overnight and persists through the morning. These features are in phase with the radial wind anomalies above the outflow layer, and propagate upwards from the surface to the mid-troposphere.

The horizontal and vertical structure of the TC diurnal cycle suggest two distinct responses to radiation: (1) a radiatively–driven response in the outflow layer, and (2) a convective response (driven by latent heating) in the boundary layer. We propose that each level of heating drives a circulation, and we proceed to discuss these qualitatively. Fig. 2.8 shows the composite mean radial–vertical wind vectors as well as the net radiative tendency for both 15:00 LT and 03:00 LT. In the afternoon, radiative heating is observed at a height of 12 km (Fig. 2.8a), which is associated with both anomalous upward motion in the region of heating as well as anomalous radial outflow near this level. Peak amplitudes of anomalous vertical motion approach 5 cm s$^{-1}$ near the center of the heating, with anomalous radial outflow on the order of 1 m s$^{-1}$ at the leading edge of the net radiative response. Below this heating, anomalous radial inflow on the order of 0.8 m s$^{-1}$ is observed. This effect is local, and only affects the winds in close proximity with the region. In the early morning, the opposite circulation is observed (Fig. 2.8b), which we interpret as radiative cooling driving
weak anomalous inflow into the TC outflow layer near a height of 12.5 km, and subsidence through this region. Peak amplitudes of anomalous downward motion of 1 cm s$^{-1}$ near the center of cooling are observed, which is slightly weaker than the daytime response. A layer of anomalous radial outflow on the order of 1 m s$^{-1}$ is also present beneath this layer at 10 km height, which also suggests a local circulation at this level.

Fig. 2.9 shows the same wind vectors along with the anomalous latent heating tendency. During the day, subsidence is observed from 50–400 km, consistent with heating in the upper troposphere at the top of the layer near 12.5 km from absorption of solar radiation. Minimum values of latent heating tendency anomalies are -0.6 K h$^{-1}$ near the storm core, and coincide with downward motion on the order of 5 cm s$^{-1}$. Anomalous radial outflow is present in the
boundary layer from a radius of 50 km to a radius of 400 km with magnitudes near 1 m s\(^{-1}\). At night, convection drives an overturning response throughout the troposphere (Fig. 2.9a). Maximum values of latent heating tendency anomalies approach 1.2 K h\(^{-1}\) at a radius of 50 km. Anomalous upward motion is observed through the region of heating for radii of 75–150 km, and from the top of the boundary layer to a height of 10 km. This anomalous upward motion has a magnitude that exceeds 5 cm s\(^{-1}\), and is associated with a deep layer of anomalous radial inflow on the order of 0.8 m s\(^{-1}\) that extends to outer radii near 400 km. Anomalous radial outflow of magnitude 0.6 m s\(^{-1}\) is observed for radii of 100–400 km and from the top of the boundary layer near 4 km height. These results suggest a wind response in the low-levels, linked to anomalous latent heating, that is a maximum near the core of the storm.

Fig. 2.10a shows the time–mean cloud fraction for the equilibrium storm. Highest cloud
fractions are observed in the eyewall region, with maximum values of 70% indicated near the RMW at 50 km from near the surface up to 8 km in height. Positive values extend upwards to 14 km height, indicating the top of the cloud layer. Cloud fractions in the upper troposphere extend from the inner core of the storm near 50 km out to 350 km in radius, illustrating the TC outflow layer. The time–mean infrared (LW) cloud flux demonstrates a maximum in this region, with values of 160 W m$^2$ at 100 km radius and 10 km height (Fig. 2.10b). Cloud flux is defined as the all–sky minus clear–sky values, with positive indicating net flux downward. The vertical gradient of the LW cloud fluxes increases from the surface

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4 The vertical gradient of the flux is proportional to the heating rate.
up to 10 km, indicating warming in the mid–troposphere, and decreases with height from 10 km into the lower stratosphere, indicating cooling near and above the cloud–top. The time–mean solar (SW) cloud flux opposes the LW cloud flux, demonstrating negative values throughout the domain (Fig. 2.10c). Minimum flux values are near -3 W m$^2$ from 2 km height up to 11 km height, which is two orders of magnitude less than the LW fluxes. A local minimum in the region of the largest cloud fraction suggests the incident beam is attenuated in the eyewall cloud via scattering. The time–mean distribution of the ice mixing ratio demonstrates large concentrations near the level of the TC outflow (Fig. 2.10d); maximum concentrations approach $2 \times 10^{-4}$ g g$^{-1}$. The maximum in ice concentration coincides with the maximum in the LW cloud flux, suggesting the distribution of ice in the upper troposphere may impact the magnitude of the LW radiative tendencies.

Fig. 2.11 demonstrates the vertical structure of the diurnal cycle for the same four fields. The composite cloud fraction anomalies demonstrate an increase in cloud concentration during the day, and a decrease at night (Fig. 2.11a). Maximum anomalies approach 10% near 14 km height from 13:00 LT to 21:00 LT, and extend downward into the TC outflow layer near 12.5 km from 15:00 LT to 21:00 LT. A similar pattern in the high–cloudiness of the TC outflow layer in real storms has been documented, with an expansion of the TC cirrus canopy observed during the day and contraction at night (e.g., Kossin, 2002). These results are consistent in timing with previous observations. A separate pattern is indicated in the mid–lower troposphere, where positive anomalies of a 7% increase in cloud fraction are observed overnight from 03:00 LT through 11:00 LT. Negative anomalies of similar magnitude are observed during the day from 13:00 LT though 1:00 LT. This pattern suggests an increase of deep convection in the eyewall overnight and suppression of convection throughout the day, consistent with the anomalous latent heating tendencies discussed previously (Fig. 2.6b). These same two regions are highlighted in the LW cloud flux anomalies (Fig. 2.11b), as well as the anomalous ice concentration (Fig. 2.11d). LW cloud flux anomalies of 10 W m$^2$ are indicated near 12.5 km height at the same time an anomalous ice concentration is indicated above the TC outflow layer at 14 km height from 13:00 LT to 19:00 LT, suggesting that the
Figure 2.11: Hovmöller diagrams for the (a) cloud fraction (\%), (b) infrared cloud radiative flux (W m\(^{-2}\)), (c) solar cloud radiative flux (W m\(^{-2}\)) and (d) ice mixing ratio (g/g). Fields are composite anomalies computed at each hour. Cloud radiative flux is defined as the all–sky minus clear–sky fluxes. Radiative fluxes are net positive downward, and fields in (b)-(d) are radial averages from 50-100 km.

diurnal cycle in high ice clouds directly impacts the LW cloud flux, and therefore the heating tendencies. The SW cloud flux mainly reflects the mean distribution, with negative values of -4 W m\(^{-2}\) at 13:00 LT from the surface up to 12.5 km height 2.11c). Positive values in solar cloud flux at night reflect the subtraction of the time–mean values.

2.3 Summary and Conclusions

The goal of this work is to quantify a diurnal signal for TCs and investigate its relationship to storm structure and intensity. A numerical experiment is conducted in an axisymmetric
framework with minimal environmental influence to explicitly identify the internal response of the storm to the diurnal cycle of radiation. Time-series analysis reveals a coherent TC diurnal signal that dominates the variance in the outflow layer and accounts for 62% of the variance in temperature in this region. At low levels, the diurnal cycle accounts for up to 25% of the variance in the radial wind field and 21% of the variance in the azimuthal wind field. A weak diurnal signal is also present in the mid-troposphere, accounting for 2% of the variance in the latent heating field. These features are all statistically significant at the 95% confidence level.

Composite analysis shows a complex response in the storm structure linked to anomalous latent heating in the boundary layer and to absorption of radiation in the TC outflow layer. In the boundary layer, a periodic overturning circulation drives anomalous upward motion in the region near the radius of maximum wind; this feature is associated with a deep layer of anomalous radial inflow. This anomalous inflow drives larger values of angular momentum from the storm environment into the inner core of the storm, which, by conservation principles, must result in an increase in tangential winds. This is consistent with the positive azimuthal wind response observed in the early morning that lags the positive response in the latent heating field by approximately 5 h. This suggests that the TC diurnal cycle favors storm intensification in the early hours of the morning and storm weakening in the late afternoon and evening.

In the outflow layer, observed disturbances are downward and outward propagating features; in the stratosphere, these disturbances have estimated vertical wavelengths of 4 km and horizontal phase speeds are 19.6 m s\(^{-1}\). In the troposphere, phase speeds are 9.8 m s\(^{-1}\). These phase speed are similar to speeds documented by both Steranka et al. (1984) and Dunion et al. (2014), who estimate values near 5–10 m s\(^{-1}\) for real storms. Comparing these results to perturbations introduced in a rotating, stably-stratified atmosphere suggests correspondence to an inertia–gravity wave response. One characteristic of inertia–gravity waves is that the phase speed and group velocity are orthogonal; in this case, downward propagating phase implies upward propagating group velocity, and thus wave energy propagates out and
away from the storm. It is unclear at this time what impact, if any, these waves have on the overall storm structure. The presence of gravity waves in TCs is an active area of research and is the subject of future study.

A hypothesis is drawn from these results that the TC diurnal cycle is a combined response from two periodic heat sources: one in the TC outflow due explicitly to radiation, and one in the boundary layer due to latent heating, which derives indirectly from radiative cooling in the upper troposphere. These heat sources each correspond to a local circulation that drives an anomalous tangential wind response and affects storm intensity. The present findings are qualitative and will be tested quantitatively in a follow-up study exploring the effects of periodic heating in an idealized vortex.

One additional result from this work is the presence of a semidiurnal signal in the power spectrum. A semidiurnal signal has been observed near the convective region in the inner core of real storms, and has been attributed to a modulation of lapse rates by the atmospheric semidiurnal tide (S₂, Kossin 2002). This model contains a damping layer in the stratosphere, which inhibits the effect of an atmospheric tide. We are thus not capable of evaluating the influence of S₂ in this model. We suggest a further evaluation of the semidiurnal signal in the numerical simulation, as well as a testing of the hypothesis, as possible avenues of future work.

The present study is limited to the configuration of the model, which only includes the NASA-Goddard longwave and shortwave radiation schemes. Future work should evaluate the robustness of these results to changes in the microphysics scheme. Features such as the amount of pristine ice in the upper troposphere may quantitatively affect the results, as this would alter the radiative tendencies, which may affect the strength of the hypothesized circulations. In addition, three-dimensional analysis is required to evaluate the role of asymmetries, as cloud fields may be overestimated in the axisymmetric framework. However, given the qualitative agreement of the results presented here and observations, we believe such changes may not significantly affect the results.
Chapter 3

BALANCED RESPONSE OF AN AXISYMMETRIC TROPICAL CYCLONE TO PERIODIC DIURNAL HEATING

3.1 Introduction

Recent observations demonstrate a clear diurnal signal in the high clouds of tropical cyclones (TCs). Although this oscillation is linked to storm structure and intensity, the impact of periodic diurnal heating in TCs is essentially unknown. A numerical study examining the TC diurnal cycle in a statistically steady-state framework shows a periodic wind and temperature response (Navarro and Hakim, 2016) (NH2016): periodic heating in the TC outflow layer from the daily cycle of radiation coincides with a local overturning circulation, with inflow toward the base of the heating, upward motion in the region of heating, and radial outflow at the top of the heat source. Gravity waves are observed in the upper troposphere and lower stratosphere, mainly in the periodic temperature response. A second periodic signal is observed in the lower troposphere, where a diurnal oscillation in the latent heating tendency field leads a periodic response in the azimuthal wind by approximately 6 h. NH2016 propose that the dynamic response to periodic heating drives a circulation in these two regions that produces the diurnal cycle in the TC wind field. Here, we use a modified Sawyer-Eliassen approach for time-varying heating to analyze the effect of periodic diurnal heating on a balanced vortex.

Most of the previous work on the TC diurnal cycle is devoted to documenting storm high-cloudiness (Browner et al., 1977, Muramatsu, 1983, Lajoie and Butterworth, 1984, Steranka et al., 1984, Kossin, 2002). A coherent diurnal signal is observed in the TC cirrus canopy, which propagates radially away from the storm center (Steranka et al., 1984, Dunion et al., 2014). This signal is consistent across ocean basins, and is a function of the local solar time.
Numerical modeling studies of the TC diurnal show a wide range of results, with some studies showing a large impact in the developing stages (Sundqvist, 1970, Hobgood, 1986, Melhauser and Zhang, 2014, Tang and Zhang, 2016), others showing impact during the mature stage (Hack, 1980, Tuleya and Kurihara, 1981, Craig, 1996, Tang and Zhang, 2016). Currently, there is no consensus in the literature on the role of the diurnal cycle of radiation on TC structure and intensity.

Recent work using an idealized, axisymmetric TC in a steady-state framework demonstrates a clear signal in the temperature, wind, and latent heating tendency fields (Navarro and Hakim, 2016, hereafter NH2016). NH2016 show that the TC diurnal cycle accounts for 62% of the variance of the temperature in the TC outflow layer, and 21% of the variance in the azimuthal wind in the lower troposphere. Composite analysis of these fields at each hour of the day reveals a cycle in storm intensity that is a maximum in the early hours of the morning and lags a periodic response in latent heating by approximately 6 h. Although the mean diurnal signal is small, anomalies at individual times exhibit large variance, with perturbation azimuthal wind values of up to 10 m s\(^{-1}\) near the radius of maximum wind (RMW). Gravity waves are observed in the upper troposphere and lower stratosphere, linked with positive temperature anomalies that arise in the TC outflow layer. NH2016 hypothesize that these periodic heat sources drive anomalous circulations in the upper and lower troposphere that drive the TC diurnal cycle.

The goal of this work is to test the NH2016 hypotheses by diagnosing the impact of periodic diurnal heating on a balanced vortex. The Sawyer-Eliassen equation has been used to compute the secondary flow induced by quasi-steady forcing in a balanced vortex (Schubert and Hack, 1982, Shapiro and Willoughby, 1982, Pendergrass and Willoughby, 2009). Solutions are a balanced, overturning response to heating in the radius–height plane for a vortex which evolves gradually in time. Periodic, as compared to steady, forcing, however, requires a different formulation in which the governing equations are linearized about the mean vortex, and describes periodic perturbations induced by periodic forcing on the mean vortex. The result is an equation for the perturbation streamfunction, which is similar to
the Sawyer-Eliassen equation except that the coefficients of the second derivatives are functions of the forcing frequency (Willoughby, 2009). For low frequency forcing, solutions are a balanced response to periodic heating that resembles the Sawyer-Eliassen solution; for high frequency forcing, solutions project onto radiating inertia–buoyancy waves.

This chapter is organized as follows. Description of the time-varying method, the vortex and heating specification are given in section 3.2. Results are shown in section 3.3, and section 3.4 provides a discussion and the conclusions.

3.2 Methods

3.2.1 Model Configuration

Using the method of Willoughby (2009), we solve for the perturbation mass–flow streamfunction induced by periodic diurnal heating on a gradient and hydrostatically–balanced vortex. A full derivation of this equation is given in Willoughby (2009); only the governing equations and the solution method are described here.

The linearized governing equations for periodic, axially-symmetric perturbations on a vortex in gradient and hydrostatic balance are (Willoughby, 2009):

\begin{align}
-i\omega v + \zeta u + Sw &= M \\
-i\omega u - \xi v + \gamma b &= -c_p\theta_0 \frac{\partial \pi}{\partial r} \\
-i\omega w - b &= -c_p\theta_0 \frac{\partial \pi}{\partial z} \\
-i\omega b + Bu + N^2 w &= Q' \\
\frac{\partial (r\rho u)}{\partial r} + \frac{\partial (r\rho w)}{\partial z} &= 0
\end{align}

where \( \omega \) is the forcing frequency. Here, \( u(r, z, t) \), \( v(r, z, t) \), and \( w(r, z, t) \) are the perturbation radial, tangential, and vertical velocities, respectively, and \( \pi (r, z, t) \) is the perturbation Exner function. The vortex has mean flow vertical vorticity, \( \zeta = \partial v_0 / \partial r + v_0 / r + f \), where \( v_0 \) is the
mean vortex azimuthal wind and $f$ is the Coriolis parameter. The vertical wind shear is given by $S = \partial v_0 / \partial z$. $M$ is the momentum forcing which here is set equal to zero. $\xi = 2v_0 / r + f$ is the inertia parameter, and $\gamma = g^{-1} (v^2 / r + f v)$ is the ratio of the mean-flow radial acceleration to gravity. The perturbation buoyancy is $b(r, z, t) = g \left[ \theta(r, z, t) - \theta_o(r, z, t) \right] / \theta_o(r, z)$, which corresponds to perturbation virtual potential temperature $\theta(r, z, t)$. $c_p$ is the specific heat at constant pressure for dry air. The radial and vertical gradients of the mean vortex buoyancy are $B = \partial b_0 / \partial r$ and $N^2 = \partial b_0 / \partial z$, where $b_0 (r, z) = g \ln (\theta_0 / 273.16)$ is the mean state buoyancy. $B$ has units of $s^{-2}$ and $N^2$ is the square of the buoyancy. $Q' = g q / c_p \theta_0$ is the diabatic perturbation buoyancy source, where $q$ is the heating rate. The mean-state air density is given by $\rho = 1000 \pi_0^c / R \rho_0 / R \theta_0$, where $\pi_0(r, z) = (p_0 / 1000) R / c_p$ is the mean Exner function computed from the pressure, $p_0(r, z)$. The gas constant is given by $R$, and $c_v$ is the specific heat of dry air at constant volume.

Solving this system of equations for $u$ and $w$ and introducing a mass-flow streamfunction $\psi$ such that

$$r \rho u = -\frac{\partial \psi}{\partial z}, \quad r \rho w = \frac{\partial \psi}{\partial r}$$

(3.2)

gives a second-order differential equation for the perturbation streamfunction (Willoughby, 2009):

$$\left( N^2 - \omega^2 \right) \frac{\partial^2 \psi}{\partial r^2} - 2B \frac{\partial^2 \psi}{\partial r \partial z} + \left( I^2 - \omega^2 \right) \frac{\partial^2 \psi}{\partial z^2}$$

$$- \left[ \frac{(N^2 - \omega^2)}{R_{\theta \rho}} - \frac{B}{H_{\theta \rho}} \right] \frac{\partial \psi}{\partial r}$$

$$- \left[ \frac{(I^2 - \omega^2)}{H_{\theta \rho}} - \frac{B}{R_{\theta \rho}} - \frac{3 \xi S}{r} + B \frac{\partial \gamma}{\partial z} - N^2 \frac{\partial \gamma}{\partial r} \right] \frac{\partial \psi}{\partial z}$$

$$= r \rho \left[ \left( \frac{\partial}{\partial r} - \frac{1}{L_{\theta}} \right) Q - \left( \frac{\partial}{\partial z} - \frac{1}{H_{\theta}} \right) (\xi M - \gamma Q) \right]$$

(3.3)

where
\[
\frac{1}{R_{\theta \rho}} = \frac{1}{r} + \frac{1}{\theta_0} \frac{\partial \theta_0}{\partial r} + \frac{1}{\rho} \frac{\partial \rho}{\partial r} \quad (3.4a)
\]

\[
\frac{1}{H_{\theta \rho}} = \frac{1}{\theta_0} \frac{\partial \theta_0}{\partial z} + \frac{1}{\rho} \frac{\partial \rho}{\partial z} \quad (3.4b)
\]

\[
\frac{1}{L_{\theta}} = \frac{1}{\theta_0} \frac{\partial \theta_0}{\partial r} \quad (3.4c)
\]

\[
\frac{1}{H_{\theta}} = \frac{1}{\theta_0} \frac{\partial \theta_0}{\partial z} \quad (3.4d)
\]

and \( I'^2 = (\zeta \xi)^2 - \gamma B \) is a modified local inertia frequency\(^1\). Equation (3.3) is similar to the Sawyer-Eliassen equation except that the coefficients of the second partial derivatives are functions of the forcing frequency, and the character of the solution changes based on the sign of the coefficients. The discriminant of equation (3.3) defines the solution’s behavior:

\[
D^4 = (N^2 - \omega^2) \left( I'^2 - \omega^2 \right) - B^2. \quad (3.5)
\]

When \( D^4 > 0 \), equation (3.3) is elliptic and the solutions are a balanced response to heating that resemble the Sawyer-Eliassen solution. For \( D^4 < 0 \), equation (3.3) is hyperbolic and solutions project onto radiating inertia–buoyancy waves. Since \( I'^2 \) is large in the storm core, and since the diurnal cycle is a low-frequency forcing (i.e., \( \omega \ll I' \)), we expect that solutions near the core of the storm will resemble the Sawyer-Eliassen solution. However, for solutions further from the center (i.e., \( \omega \geq I' \)) solutions will manifest as gravity waves.

Solutions are obtained using the Lindzen and Kuo (1969) algorithm for the specified forcing and initial vortex structure. The periodic forcing is sinusoidal, and oscillates at a frequency of \( \omega = \frac{2\pi}{\tau} \) where \( \tau = 24 \) h. The maximum amplitude of the heating occurs at \( t = 0 \), and the minimum at \( t = 12 \) h\(^2\). Unless otherwise noted, all solutions are shown at time \( t = 0 \). The domain extends from the surface up to 30 km in the vertical and from the

\(^1\)The \( I' \) and \( \gamma \) terms arise due to the derivation in height versus pressure coordinates.

\(^2\)The real part of the solutions are in phase with the maximum amplitude in heating; imaginary parts are one quarter out of phase with the maximum amplitude (a lag of \( \frac{\pi}{2} \), or 6 h)
vortex center to 1500 km in the horizontal. Vertical and horizontal grid resolutions are 1.5 and 2 km, respectively. There is no axially symmetric flow into the surface, across the center of the vortex, or out of the top of the domain. A sponge layer is imposed at the domain top to prevent reflection of gravity waves back down toward the surface. At the right edge of the domain $\frac{\partial \psi}{\partial r} = 0$, and flow is strictly horizontal. There is no friction. The vortex is initialized from an idealized environmental sounding in which the temperature decreases from 300 K at the surface to 200 K at the tropopause (15 km), and is isothermal at 200 K in the stratosphere.

The mean vortex is the “idealized vortex” described in Pendergrass and Willoughby (2009) and Willoughby (2009). This configuration allows for storm parameters such as size, strength, and the location of the RMW to be easily analyzed and changed. Winds decreases linearly with height from the surface to the top of the vortex at 18.5 km (Fig. 3.1). Maximum values of 50 m s$^{-1}$ are observed at the surface at a radius of 30 km, denoting the location of the radius of maximum wind (RMW). The RMW is tilted in the vertical, and slopes outward from the surface to a radius of 46 km at the vortex top. Inside the RMW, a power law with an exponent equal to 1 is used, such that winds increase linearly with radius; outside the RMW, winds decrease exponentially with an $e$-folding distance of 300 km. The transition zone in the region of the RMW is 20 km-wide, and is a weighted sum of the inner and outer wind profiles (Willoughby et al., 2006).

Fig. 3.2 shows the discriminant of Equation 3.3, which defines the boundary between elliptic and hyperbolic solutions. Positive values are observed in the core of the storm, with largest values in a narrow region from 0-60 km in radius and up to 19 km in height. This reflects the large gradient of local inertial frequency in this region, and indicates elliptic solutions similar to the Sawyer-Eliassen solution. Above 19 km, and for radii beyond 350 km at the surface, values of the discriminant are negative, which indicates that solutions in this region will manifest as inertia–buoyancy waves. The zero contour is the boundary between the two types of solutions, and gradually slopes downward from the upper troposphere to the surface with increasing radius from storm center.
Figure 3.1: The mean vortex azimuthal wind (m s\(^{-1}\)). The solid red line indicates the location of the radius of maximum wind.

3.2.2 Heating Distributions

The hypotheses introduced in NH2016 are derived from a simulation produced in axisymmetric Cloud Model 1 (CM1), which is a non-hydrostatic cloud model that simulates the effects of shortwave and longwave radiation (Bryan and Rotunno, 2009b). Fig. 3.3a-b. show the CM1 diurnal cycle hypotheses, which are comprised of two distinct, periodic heat sources. These heat sources are anomalies from the time-mean values of net radiative tendency and latent heating tendency, averaged over the steady–state portion of simulation (300 days). At 03:00 local time (LT), a maximum in the latent heating tendency is observed in the lower
Figure 3.2: The discriminant \((s^{-4})\) of eq. 3.3 for the initial vortex shown in Fig. 3.1.

troposphere near the radius of maximum wind\(^3\) (Fig. 3.3a.). Values of the latent heating tendency are 1.2 K h\(^{-1}\) at a radius of 50 km. This heating is associated with a deep layer of radial inflow, which extends from 50-400 km in radius and exhibits an average magnitude of 1 m s\(^{-1}\). Upward motion is observed in the region of heating, with values near 5 m s\(^{-1}\). Flow turns outward at 10 km height and exits the core of the storm at this level. At 15:00 LT, radiative heating warms the upper troposphere near the level of the TC outflow (Fig. 3.3b.). This heating is a maximum at 12.5 km height and exhibits a magnitude of 12 K day\(^{-1}\). Upward motion is observed near the center of heating, with radial inflow (outflow) near 1 m s\(^{-1}\) observed below (above) the heating. In both cases, heating and the associated circulations

\(^3\)The average location of the radius of maximum wind for the CM1 simulation is 53 km.
Figure 3.3: The hypotheses presented in Navarro and Hakim (2016), showing (a) the latent heating tendency at 03:00 LT and (b) the net radiative tendency at 15:00 LT for the idealized, steady-state CM1 simulation. These fields are composite anomalies from the time-mean values. The composite anomalous radial-vertical wind vectors at each time are plotted for reference.

are periodic with respect to the diurnal cycle.

Fig. 3.4 shows the idealized versions of these two heat sources for use in the Willoughby (2009) time-varying model. The heating field described in NH2016 is partitioned into two separate configurations to determine the impact of periodic diurnal heating in each layer. The CM1 latent heating tendency is given by a low level heating maximum with a center at 75 km at the surface (Fig. 3.4a). This heating has a width of 50 km and a height of 7 km. The magnitude of the diabatic forcing is scaled for consistency with the latent heating tendency in CM1. This heating is well inside the boundary defined by the discriminant, which suggests
elliptic solutions. For the CM1 radiative forcing, a heating maximum is placed in the upper troposphere at a height of 10.5 km (Fig. 3.4b). The center of this heating is at 100 km radius, and it exhibits a width of 75 km in the radial direction. Again, values are scaled for consistency with the CM1 simulation. The heating is in close proximity to the boundary given by the discriminant, and suggests that solutions may involve a combination of elliptic and hyperbolic solution types.
3.3 Results

3.3.1 Lower Tropospheric Forcing

Periodic forcing near the storm core induces an overturning circulation throughout the lower troposphere (Fig. 3.5a). A streamfunction dipole straddles the heat source, with positive values indicating anticyclonic circulation, and negative values indicating cyclonic circulation. Radial-vertical wind vectors demonstrate a deep layer of radial inflow from the surface up to 2 km that extends from 400 km in the outer storm environment to 75 km in the storm core. A shallow layer of radial outflow from 25-50 km is indicated at the surface, with convergence of winds near the base of the heat source at 75 km. Upward motion is observed in the region of heating, with diverging winds near 5 km height, and radial outflow from the region of heating to 400 km in radius. This circulation resembles what one expects from the Sawyer-Eliassen solution, as expected based on the positive values of the discriminant of Eq. 3.3. Magnitudes of the radial inflow and outflow are largest near the base of heat source, with perturbations near 1 m s\(^{-1}\). The CM1 radial-vertical wind vectors also indicate an overturning circulation (Fig. 3.5b). Radial inflow on the order of 1 m s\(^{-1}\) is observed throughout the lower troposphere, similar to the diagnostic solution. The flow turns upwards and outwards near the storm core at a radius of 75 km, with magnitudes of upward motion exceeding 5 m s\(^{-1}\) near the center of the heating at 50 km. Magnitudes of radial outflow are 0.5 m s\(^{-1}\) at a height of 10 km and from radii between 100-300 km. A region of relatively strong radial inflow is observed near a height of 13 km, which is absent in Fig. 3.5a. With the exception of this inflow layer, the CM1 circulation closely resembles the diagnostic solution, and suggests that low-level periodic forcing near the storm core dominates the tropospheric response.

The perturbation azimuthal wind for the diagnostic solution shows a low-level maximum outside the RMW (Fig. 3.6a). This field lags the maximum in streamfunction by 6 h. Perturbation amplitudes of 4 m s\(^{-1}\) are observed at the surface near a radius of 100 km and tilt upward with height. Negative values of 2 m s\(^{-1}\) are observed inside the eye from 0-50 km,
Figure 3.5: (a) The vector winds from the Willoughby (2009) perturbation streamfunction (m s$^{-2}$) and the perturbation streamfunction (kg s$^{-1}$) for the low-level forcing in Fig. 3.4a. and (b) the CM1 radial-vertical circulation at 03:00 LT.

as well as outside the RMW in the mid-troposphere from 2 to 12 km in height. The CM1 composite azimuthal wind anomalies shows a similar structure (Fig. 3.6b). This response is observed 6 h after the onset of the anomalous positive latent heating tendency, similar to the diagnostic solution. A low-level maximum of 0.75 m s$^{-1}$ is observed from 50-100 km in radius, and up to 6 km in height. This anomaly extends through the TC boundary layer from 50 km to 400 km in the exterior storm environment. Negative values of up to 0.5 m s$^{-1}$ are observed both inside the eye, as well as throughout the mid to upper troposphere. These results are consistent with the results in Fig. 3.6a, and suggest the low-level azimuthal wind accelerates in response to low-level periodic forcing. Azimuthal wind maxima just outside
3.3.2 Upper-Level Forcing

Periodic diurnal heating near the top of the storm produces a localized overturning response (Fig. 3.7a). A maximum in streamfunction is observed at a radius of 175 km and a height of 10 km, with a magnitude of 6x10^7 kg s\(^{-1}\). Radial-vertical wind vectors demonstrate that air is moving inward toward the base of the heat source at 8 km height, upward through the region of heating, and outward near 12 km height. Magnitudes of these vectors are near 0.5 m s\(^{-1}\) in the horizontal direction and 5 cm s\(^{-1}\) in the vertical direction. For radii of
300-400 km, alternating positive and negative anomalies from the surface up to the lower stratosphere indicate a gravity wave response. The CM1 response is mainly confined to the upper troposphere (Fig. 3.7b): a layer of relatively strong radial outflow is observed near 15 km height, with magnitudes exceeding 1 m s$^{-1}$. Weak radial inflow and convergence are observed beneath this layer at a radius of 100 km, which suggests a local circulation in response to upper–level heating. This is consistent with the result in Fig. 3.7a. For the lower troposphere, a broad, cyclonic circulation reflects the cooling phase of the tropospheric forcing described previously, which dominates the low–level CM1 response.

The perturbation azimuthal wind for the upper-level forcing exhibits a quadrupole in
structure (Fig. 3.8a). This response is mainly confined to the mid to upper troposphere. Positive values of $u_\theta$ are observed from 50–300 km in radius and 5–10 km in height, with negative values observed from 50–400 km in radius and 10–14 km in height. Inside the RMW the sign changes, with negative values observed from 0–50 km in radius and 5–10 km in height, and positive values from near 20-50 km in radius and from 10–12 km height. Maximum magnitudes of these perturbations are 0.5 m s$^{-1}$ (-0.5 m s$^{-1}$) for positive (negative) perturbations. A weak positive anomaly in the lower troposphere is observed from 2–5 km height at a radius of 25 km, indicating a possible low-level response. Comparing this result to the corresponding CM1 azimuthal wind anomaly shows a similar structure in the upper–troposphere (Fig. 3.8b). Positive values are indicated from 50–250 km in radius and 6–12 km in height, and
negative values are present from 50–400 km in radius and 12–16 km height. Inside the RMW, negative values are observed from 0–50 km in radius and 2–10 km in height, and positive values from 25–50 km in radius and 12–15 km in height. There is a close correspondence between the diagnostic solution and the CM1 results, which suggests that upper–level periodic heating is driving the CM1 azimuthal wind response in this region. In the lower troposphere, negative anomalies near the RMW and throughout the TC boundary layer correspond to the cooling phase of the CM1 low-level forcing (Fig. 3.6b); however, positive values near the surface from 15-50 km radius are consistent with the positive anomalies observed from 2–5 km in Fig. 3.8a.

### 3.4 Summary and Conclusions

Here, a modified Sawyer-Eliassen equation for time-varying forcing is applied to diagnose the role of periodic diurnal heating on a balanced vortex. Following the work of NH2016, two regions of periodic diurnal heating are analyzed: (1) upper-tropospheric heating near the TC outflow associated with the absorption of solar radiation and (2) lower-tropospheric heating due to latent heat release from convection. These distributions are considered separately to determine their relative impact on the vortex circulation. Time-varying heating requires a specific formulation where the governing equations are linearized about the mean state (Willoughby, 2009); the result is an equation for the time-varying, axially symmetric, circumferential component of the vorticity, where the solutions describe periodic perturbations on the mean vortex. Results for low-frequency forcing in the storm core approach the Sawyer–Eliassen solution, while solutions further in the exterior environment project onto radiating inertia–buoyancy waves.

For both upper–level and lower–level periodic forcing, comparison between the diagnostic solution the CM1 numerical simulation in NH2016 demonstrates a similar response. For the diagnostic solution, periodic diurnal heating in the storm core drives an balanced, overturning circulation throughout the lower to mid troposphere. This is associated with an acceleration of the low-level azimuthal wind 6 h after the maximum in streamfunction. Periodic heating
in the upper troposphere induces a local overturning response in the region of heating, which 
manifests as inertia–buoyancy waves in the storm environment. In both cases, the CM1 
composite azimuthal wind anomalies closely match the diagnostic solution, and suggests 
that periodic diurnal heating is driving the CM1 diurnal cycle in intensity. Partitioning of 
the CM1 heating field into two parts demonstrates that low-level periodic heating dominates 
the lower tropospheric response, while the response upper–level forcing is mainly confined to 
the upper-troposphere.

The similarity of the structure between the diagnostic solution and the CM1 diurnal cycle 
suggests that the axisymmetric TC diurnal cycle in CM1 is the combined response from two 
periodic heat sources. These heat sources are out-of-phase with each other, indicating that 
upper–level radiative heating may be indirectly related to the low–level latent heat release. 
Since the Willoughby (2009) method requires prior knowledge of the heating field, and does 
not explicitly solve for the effects of radiation or microphysics, we are unable to investigate the 
relationship between these two features in the current model. Investigation of the coupling 
between the upper–level and lower–level response in a numerical simulation is one possible 
avenue of future work.

These results depend on the chosen initial vortex, so that factors such as the mean vortex 
azimuthal wind, the location of the RMW, and the decay length of the outer wind profile 
al contribute to the location of the discriminant boundary. As this boundary changes, 
the impact of periodic diurnal heating changes, so that solutions may project more on a 
radiating wave response versus a balanced response, and vice versa. This implies that the 
TC diurnal cycle in nature may vary based on the mean state of the underlying vortex, 
and most importantly, vortex intensity. Future work will test the sensitivity of the results 
presented here to changes in the discriminant boundary, and will compare these solutions to 
the variability produced in the CM1 numerical simulation.
Chapter 4

THE SENSITIVITY OF THE AXISYMMETRIC TROPICAL CYCLONE DIURNAL CYCLE TO THE LENGTH OF THE DIURNAL PERIOD AND THE VORTEX INTENSITY

4.1 Introduction

Observations of the tropical cyclones (TC) diurnal cycle in nature indicate a clear signal in the TC cirrus canopy. Although this signal has been previously linked to storm structure and intensity, the variance of the TC diurnal cycle is not well understood. Using an axisymmetric model simulation in a statistically steady-state framework, Navarro and Hakim (2016) demonstrate the TC diurnal cycle is a forced response to periodic diurnal heating. Specifically, periodic radiative heating in the upper troposphere is indirectly linked with a cycle of latent heat release in the lower troposphere, which induces a response in storm intensity. Navarro et al. (2016) diagnose the impact of periodic diurnal heating on a balanced vortex and demonstrate that solutions dependent on the location of the heating; periodic heating near the core of the storm produces a balanced response similar to the Sawyer–Eliassen solution, while periodic heating in the storm environment mainly manifests as inertia–buoyancy waves. The boundary between these two type of solutions is a function of the forcing frequency, as well as the initial structure of the vortex (Willoughby, 2009). Here, we modify both the length of the diurnal period and the vortex intensity to evaluate the sensitivity of the Navarro et al. (2016) solutions.

Most observational studies document a daily oscillation in storm high–cloudiness (Browner et al., 1977, Muramatsu, 1983, Lajoie and Butterworth, 1984, Steranka et al., 1984). The areal extent of the cirrus canopy expands during the day, and contracts at night (Kossin, 2002). Composite analysis reveals that the TC diurnal cycle is an outward propagating signal
that is a function of the local solar time (Steranka et al., 1984, Dunion et al., 2014). Anomalous cold infrared brightness temperatures form near the core of the storm overnight, and propagate radially outward in the early hours of the morning (Dunion et al., 2014). Outward propagation speeds of 10-15 m s$^{-1}$ are observed for tropical storms and 2 m s$^{-1}$ for hurricanes (Steranka et al., 1984, Dunion et al., 2014). Numerical studies of the TC diurnal cycle demonstrate a variety of results, with some showing an increased growth rate in development (Sundqvist, 1970, Hobgood, 1986, Melhauser and Zhang, 2014, Tang and Zhang, 2016), an earlier onset of storm intensification (Hack, 1980, Tuleya and Kurihara, 1981), and higher overall maximum intensity (Craig, 1996). While these studies hypothesize that radiative cooling at night contributes to the observed changes, there is currently no agreement on the impact or the mechanism of the daily cycle of radiation in the lifetime of the TC.

A recent study using a long, statistically steady-state simulation of an axisymmetric TC quantifies the variance explained by the diurnal cycle and shows that it accounts for 62% of the variance in the temperature field near the TC outflow layer and 28% of the variance in the radial wind in the lower troposphere (Navarro and Hakim, 2016). Upper-level periodic heating associated with the absorption of solar radiation coincides with a local overturning response in the radial-vertical wind field, which projects mainly onto an inertia-buoyancy wave response in the temperature field. Low-level periodic heating associated with latent heat release from convection coincides with a deep layer of radial inflow and convergence of winds near the core of the storm, which precedes a periodic cycle in storm intensity by 6 h. Magnitudes of azimuthal wind anomalies exhibits large variance, with perturbations as large as 10 m s$^{-1}$. Using the modified Sawyer-Eliassen method of Willoughby (2009), idealized versions of these heating distributions are diagnosed for a balanced vortex, demonstrating that periodic diurnal heating produces a similar radial, vertical, and azimuthal wind response for both upper-level and low-level periodic forcing (Navarro et al., 2016).

The goal of this work is to diagnose the sensitivity of the results of Navarro et al. (2016) to changes in the forcing frequency and the mean vortex intensity. Previous work has shown that the response of a balanced vortex to periodic heating demonstrates a wide range of results
(Willoughby, 2009). For heating at the radius of maximum wind (RMW), low–frequency forcing produces elliptic solutions which resemble the Sawyer–Eliassen solution, while high frequency forcing produces hyperbolic solutions that project onto radiating inertia–buoyancy waves. The character of these solutions is a function of the radial distance from storm center, and is determined based relationship between the static stability, the radial gradient of the buoyancy, the local inertial frequency, and the period of the forcing. Using the framework of Willoughby (2009), we quantify the dependence of the TC diurnal cycle to the length of heating phase and determine the impact of the vortex intensity.

The remainder of this paper is organized as follows. Description of method is given in section 4.2. Section 4.3 describes the sensitivity of results to the length of the diurnal period, and section 4.4 shows the sensitivity of results to changes in the vortex intensity. Section 4.5 provides a discussion and the conclusions.

### 4.2 Methods

The model configuration and the heating distributions are the same as Navarro et al. (2016), but for different values of the forcing frequency and mean vortex intensity. In this method, the coefficients of the second derivatives are functions of the forcing frequency, and the character of the solution changes based on the following equation:

$$D^4 = (N^2 - \omega^2) (I'^2 - \omega^2) - B^2.$$  \hspace{1cm} (4.1)

Eq. 4.1 is the discriminant of a diagnostic equation for the perturbation mass flow streamfunction induced by periodically varying heating (Willoughby, 2009). $N^2 = \partial b_0 / \partial z$ is the vertical gradient of the buoyancy, where $b_0 (r, z) = g \ln (\theta_0 / 273.16)$ is the mean state buoyancy and $\omega$ is the forcing frequency. $I'^2 = (\zeta \xi)^2 - \gamma B$ is a modified local inertia frequency, where $\zeta = \partial v_0 / \partial r + v_0 / r + f$ is the mean flow vertical vorticity, with mean vortex azimuthal wind $v_0$ and Coriolis parameter $f$, $\xi = 2v_0 / r + f$ is the inertia parameter, and $\gamma = g^{-1} (v^2 / r + fv)$ is the ratio of the mean-flow radial acceleration to gravity. $B = \partial b_0 / \partial r$
is the radial gradient of the mean vortex buoyancy. When $D^4 > 0$, Eq. 4.1 is elliptic and solutions resemble the Sawyer–Eliassen solution. When $D^4 < 0$, Eq. 4.1 is hyperbolic and solutions are radially propagating inertia–buoyancy waves. This equation illustrates that the response to periodic heating will vary based on the value of the forcing frequency. $I$ and $B$ are strong functions of radius, such that for a given forcing frequency the sign of $D^4$ will vary based on the radial distance from the storm center. Since $N^2$ and $B$ are determined by the initial temperature profile, we adjust the forcing frequency as well as the inertial frequency through the mean vortex intensity to investigate the sensitivity of the Navarro et al. (2016) solutions. We will refer to solutions using this method as the “diagnostic” solutions.

Results from the diagnostic solution are compared to two idealized TC simulations produced in Cloud Model 1 (Bryan and Rotunno, 2009b, CM1). These simulations are the same as Navarro and Hakim (2016), except that the length of the diurnal period is modified. This adjustment is only reflected in the incoming radiation; the Coriolis parameter (and therefore the rotation rate of the earth) remains fixed. Simulations are produced for 100 days, which after a 40 day spin–up period reach statistical equilibrium with the environment. Solutions represent the average over the steady–state portion of the simulation, which equates to a sample size of 240 days for a diurnal period of 6 h, and 20 days for a diurnal period of 72 h. “Daytime” for these simulations is defined as noon local time, and “nighttime” as midnight local time, which provides ease in comparison between the heating fields for the 6 h and 72 h simulations.

Fig. 4.1 shows the discriminant for five different periods of forcing ranging from 6 h to 72 h. As the length of the diurnal period increases, the boundary between elliptic and hyperbolic solutions expands from the inner core of the storm to towards larger radii and height. For a period of 6 h, the boundary extends from 16 km height down to 150 km at the surface (Fig. 4.1a). Values inside the boundary are positive, indicating that solutions for heating in this region are elliptic. Elsewhere on the domain the discriminant is negative, demonstrating that solutions in this region are hyperbolic. The discriminant boundary moves outward slightly for a period of 12 h, extending up to 17 km height and 220 km in radius (Fig.
4.1b), and a larger jump is observed for a period of 24 h, with positive values extending out to 20 km height and 400 km in radius (Fig. 4.1c). For periods of 48 h and 72 h, the boundary between elliptic and hyperbolic solutions on the domain vanishes, indicating that all solutions are a balanced response to periodic heating, and will resemble the Sawyer–Eliassen solution (Fig. 4.1d-e).

Fig 4.2 shows the discriminant boundary for variations in the mean vortex intensity\(^1\). For a weak vortex of 5 m s\(^{-1}\), the discriminant boundary extends from near 16 km height down to 150 km at the surface, remaining near the core of the storm. As the intensity increases, the discriminant boundary expands upward and outward, with values near the surface extending from 150 km in radius to beyond 300 km for intensities above 20 m s\(^{-1}\). In the vertical, the discriminant boundary expands from 16 km to 19 km, and gradually slopes downward and outward as the intensity increases. This suggests that near the core of the storm, all vortices will exhibit a balanced response to periodic heating (i.e., elliptic solutions). However, weak vortices will exhibit an inertia–buoyancy wave response to heating in the exterior environment, with stronger storms exhibiting less and less of a gravity wave response to heating in this region. As the vortex intensity increases, heating near the core of the storm produces more Sawyer–Eliassen like solutions, with stronger storms exhibiting a more balanced response to heating. For heating in the lower troposphere near the core of the storm, we expect all solutions to resemble the Sawyer–Eliassen solution.

### 4.3 Sensitivity to the Length of the Diurnal Period

#### 4.3.1 Upper-Level Forcing

The perturbation streamfunction response for upper–level periodic forcing demonstrates a wide range of solutions (Fig. 4.3). For a period of 6 h, inertia–buoyancy waves are observed throughout the domain (Fig. 4.3a). Positive values are observed near 10 km height at the level of heating and emanate both above and below this level, with alternating positive

\(^{1}\)The period of the forcing for these experiments is fixed at 24 h.
Figure 4.1: The discriminant ($s^{-4}$) of the diagnostic solution for periods of forcing of (a) 6 h, (b) 12 h, (c) 24 h, (d) 48 h, and (e) 72 h. The zero contour (bold line) represents the boundary between elliptic and hyperbolic solutions.
and negative values extending into the upper and lower troposphere. The perturbation streamfunction response for periods of 12 h and 24 h also exhibits gravity waves (Fig. 4.3b-c). However, as the length of the period increases, solutions for upper–level forcing become less wavelike in nature and transition to a more balanced response. For periods of 48 h and 72 h, a single positive gyre is evident near 10 km height, demonstrating an overturning response. Positive values indicate anticyclonic circulation, with air moving inward towards the base of the heating, upward in region of heating, and outward at the top of the heat source.

The perturbation azimuthal wind response induced by upper–level periodic forcing also shows a range of responses (Fig. 4.4a-e). This field is one quarter period out of phase with the maximum in streamfunction, which implies a lag time of 2 h for a period of 6 h, 3 h for a period of 12 h, etc. For a period of 6 h, wave solutions are observed throughout the domain, particularly in the upper troposphere (Fig. 4.4a). Maximum positive perturbations
Figure 4.3: The perturbation streamfunction (kg s\(^{-1}\)) for upper–tropospheric heating for a period of (a) 6 h, (b) 12 h, (c) 24 h, (d) 48 h, and (e) 72 h.
approach 0.3 m s$^{-1}$ at the level of heating, and negative perturbations approach -0.3 m s$^{-1}$ from 10–20 km height. Wave solutions are also observed for a period of 12 h, with magnitudes similar to the 6 h case (Fig. 4.4b). As the length of the period increases, solutions become more symmetric in structure, exhibiting a quadrupole structure near the upper levels of the storm (Fig. 4.4c-e). For a period of 72 h, positive values are observed below the heat source from 50–300 km in radius and from 5-10 km height, and negative values are observed inside the RMW from 0–50 km. Above the heating, negative perturbations are observed from 50–400 km in radius and from 12–15 km height, and positive values are observed inside the RMW from 0-50 km radius and 6-10 km height. This structure is consistent across solutions from a period of 12 h to a period of 72 h, and demonstrates a balanced response to heating. The magnitude of these perturbations increases with longer periods, with absolute values of 1.2 m s$^{-1}$ for a period of 72 h. With the exception of the 6 h period, all solutions show weak positive anomalies extending from the base of the heating down to the surface near 25 km, indicating a possible low-level connection.

Comparing the 6 h and 72 h diagnostic solutions to the daytime circulations in CM1 shows mainly a similar upper–level response (Fig. 4.5). For a period of 6 h, a maximum in radiative heating of 15 K day$^{-1}$ is observed at a height of 12.5 km, which coincides with the mean level of the TC outflow layer. Upward motion is observed in the region of heating from 50–150 km, with maximum values of 5 cm s$^{-1}$. Above the heating, radial inflow is observed at a height of 13 km both inside the RMW at 53 km and in the exterior environment; however, no other coherent circulation is observed. For a period of 72 h, the daytime circulation shows a well-defined response in the radial–vertical wind vectors (Fig. 4.5b). Maximum values of radiative heating are 16 K day$^{-1}$ from 50–150 km and 12.5 km height, with upward motion in the region of heating near the RMW and radial outflow from 75–400 km in radius and 10–15 km in height. Magnitudes of upward motion are near 25 cm s$^{-1}$, which is 5 times larger than the upward motion observed for the 6 h diurnal period. Below this level, a deep layer of mid–level radial inflow is observed from 50–400 km in radius and from 5–10 km height, with magnitudes near 3 m s$^{-1}$. These solutions correspond to the localized overturning response
Figure 4.4: The perturbation azimuthal wind (m s$^{-1}$) for a period of (a) 6 h, (b) 12 h, (c) 24 h, (d) 48 h, and (e) 72 h. Fields lag the perturbation streamfunction response in Fig. 4.3a-e. by one quarter period.
Figure 4.5: The daytime CM1 net radiative tendency (shading, K day\(^{-1}\)) and radial-vertical velocity vectors (black arrows, m s\(^{-1}\)) for a diurnal period of (a) 6 h and (b) 72 h. Vertical velocity vectors are scaled for appearance.

of the diagnostic solution for a period of 72 h (Fig. 4.3e), and suggest a balanced response to upper-level heating.

Comparing the corresponding daytime azimuthal wind anomalies from the CM1 simulations to the diagnostic solutions shows a close correspondence of results (Fig. 4.6a)\(^2\). Alternating positive and negative anomalies are observed throughout the upper troposphere and lower stratosphere for the 6 h period, with absolute values near 0.2 m s\(^{-1}\). In the lower troposphere, the azimuthal wind anomalies are mostly negative, with magnitudes near −0.1 m s\(^{-1}\).

The magnitude and structure of these results are similar to the diagnostic solution in Fig. 4.4a, and suggest an inertia–gravity wave response to upper-level periodic heating. The composite azimuthal wind anomalies for the 72 h diurnal period show a well–defined upper and

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\(^2\)Since the timescale for geostrophic adjustment in CM1 is approximately 9 h, which is greater than the modified diurnal period for this experiment, we utilize a lag of 3 h here to analyze the perturbation azimuthal wind response.
lower tropospheric response (Fig. 4.6b). Positive azimuthal wind anomalies are observed near 10 km height and from 50–300 km in radius, which extend down to the surface near the RMW. At the surface, negative azimuthal wind anomalies are observed inside the eye from 0–25 km, as well as outside the RMW from 50–400 km in radius and throughout the low–mid troposphere. In the eye, positive anomalies are observed from 20–50 km in radius and from 10–15 km height, with negative anomalies extending out into the exterior environment. Absolute values of the azimuthal wind perturbations are near 3 m s$^{-1}$, which are 30 times larger than the 6 h diurnal period. These results correspond to the diagnostic solution for a period of 72 h, and demonstrate both a similar magnitude and structure in the azimuthal wind response (Fig. 4.4e).
4.3.2 Lower-Level Forcing

For low–level periodic heating, the perturbation streamfunction exhibits mostly balanced solutions (Fig. 4.7a-e). For a period of 6 h, positive values are indicated from 100–300 km in radius and from 0–5 km in height (Fig. 4.7a). This shows radial inflow towards the base of the heating near 75 km and upward motion in the region of heating from 0–5 km height. Negative values are observed in the exterior environment from 350–400 km, as well as into the upper troposphere and lower stratosphere, indicating wave–like behavior. As the period of heating increases, solutions more closely resemble the Sawyer–Eliassen response (Fig. 4.7b-e). For a period of 72 h, a broad overturning circulation is indicated throughout the domain, with a deep layer of radial inflow observed from 75–400 km in radius that extends from the surface up to 2 km height (Fig. 4.7e). Weak negative perturbations are indicated from 0–50 km, indicating a dipole response. While solutions for periods longer than 12 h demonstrate mostly elliptic solutions throughout the lower–mid troposphere, solutions for periods of 6 h and 12 h are a combination of elliptic and hyperbolic solutions, indicating a mixture of both a balanced response and inertia–buoyancy waves.

The perturbation azimuthal wind for low–level forcing indicates acceleration of winds near the region of heating (Fig. 4.8). For a period of 6 h, positive azimuthal wind perturbations are indicated from 75–200 km, tilting upwards to 5 km height (Fig. 4.8a). Maximum magnitudes are near 1 m s\(^{-1}\). Negative values are observed near the surface from 0–50 km in radius and throughout the mid–troposphere, with magnitudes near 0.5 m s\(^{-1}\). Wave–like behavior is evident in the outer domain near 300–400 km in radius, and in the lower stratosphere at 20 km height. As the period of forcing increases, magnitudes of the perturbation azimuthal wind increase by an order of magnitude, with values ranging from 1 m s\(^{-1}\) for a period of 12 h and 10 m s\(^{-1}\) for a period of 72 h (Fig. 4.8b-e). The structure of the wind field is consistent across periods, suggesting a balanced response to low–level heating.

Comparing these results to the 6 h and 72 h nighttime circulations in CM1 demonstrates relatively weak low–level responses (Fig. 4.9). For a period of 6 h, a shallow positive latent
heating anomaly is observed at 50 km radius and extends up to 5 km in height, with a magnitude of 0.5 K h$^{-1}$ (Fig. 4.9a). Negative anomalies of $-0.1$–$-0.3$ K h$^{-1}$ are observed throughout the lower troposphere, with the radial–vertical wind vectors indicating weak subsidence and divergence from 50–100 km radius and 5–10 km height. In contrast, the 72 h diurnal cycle exhibits a well-defined low–level response (Fig. 4.9b). Positive latent heating anomalies are observed near the RMW with a magnitude of 3 K h$^{-1}$, which coincide with radial inflow on the order of 3 m s$^{-1}$ near the surface from 50–100 km. Compared to the diagnostic solution, this inflow layer is more radially confined, remaining in the inner-core region of the storm. Upward motion is observed in the region of heating near 50 km radius,
Figure 4.8: The perturbation azimuthal wind (m s$^{-1}$) for a period of (a) 6 h, (b) 12 h, (c) 24 h, (d) 48 h, and (e) 72 h. Fields lag the perturbation streamfunction response in Fig. 4.7a-e. by one quarter period.
Figure 4.9: The nighttime CM1 net radiative tendency (shading, K day$^{-1}$) and radial-vertical velocity vectors (black arrows, m s$^{-1}$) for a diurnal period of (a) 6 h and (b) 72 h. Vertical velocity vectors are scaled for appearance.

and mid-level radial outflow extends from this region out to 400 km in radius. Upper-level radial inflow on the order of 1 m s$^{-1}$ is observed from 50–150 km in radius and 6–10 km height, which corresponds with radiative cooling (not shown). While the 72 h solution does differ from the diagnostic solution, this overturning circulation does resembles a Sawyer–Eliassen-like response in the low levels.

Fig. 4.10a shows the CM1 azimuthal wind anomalies for the 6 h diurnal period. Negative anomalies extend throughout the domain, with magnitudes near $-0.2$ m s$^{-1}$ observed in the upper troposphere from 25–50 km radius and 10–15 km height, and in the lower troposphere from 25–100 km in radius from the surface up to 10 km in height. Positive anomalies are indicated from 50–150 km in radius and from 7–10 km height, which reveals an alternating negative-positive-negative pattern and a possible wave-like response. The 72 h diurnal period shows a clear response in the lower troposphere (Fig. 4.10b). Positive azimuthal wind
Figure 4.10: The CM1 azimuthal wind anomalies (m s$^{-1}$) for a diurnal period of (a) 6 h and (b) 72 h. The top lags the nighttime minimum in heating by 3 h, and the bottom panel lags the minimum by 6 h.

anomalies of 1.5 m s$^{-1}$ are observed near the surface from 50–100 km in radius, which extend upward into the mid–troposphere. Inside the RMW, negative azimuthal wind anomalies are observed from 0–40 km in radius, which slope upward and outward towards the mid–troposphere. This circulation resembles a combination of both the upper–level and the lower–level azimuthal wind responses from the diagnostic model, and indicates an acceleration of the low-level azimuthal wind in the region of heating (Fig. 4.10b). Magnitudes of the azimuthal wind for the 72 h diurnal cycle exceed the 6 h diurnal period by an order of magnitude, which is also consistent with the diagnostic results.

4.4 Sensitivity to the Mean Vortex Intensity

Fig. 4.11 demonstrates the sensitivity of the perturbation azimuthal wind in the diagnostic solution for low–level periodic heating as a function of the mean vortex intensity. The
perturbation azimuthal wind increases nearly linearly, with higher perturbation amplitudes indicated for stronger vortices. For weak vortices with intensities of 10–30 m s$^{-1}$, azimuthal wind perturbations vary from 2–4 m s$^{-1}$, while vortices with mean intensities of 60–70 m s$^{-1}$ exhibit azimuthal wind perturbations of 5 m s$^{-1}$ or more. This result is similar to those of Schubert and Hack (1982) and Shapiro and Willoughby (1982), who demonstrate that the local efficiency of heating is larger in regions of higher inertial stability, leading to stronger tangential accelerations. This indicates that the magnitude of the diurnal signal increases for stronger vortices, and implies that the response varies throughout the lifetime of the storm, with possible weaker diurnal signals in the development and early stages of the TC and larger signals in the mature state of the TC.
4.5 Summary and Conclusions

Following the work of Navarro et al. (2016), the sensitivity of the TC diurnal cycle to variations of the diurnal frequency and the mean storm intensity is determined on a balanced vortex. Using a modified Sawyer–Eliassen approach for time–varying heating, results quantify the dependence of the TC diurnal cycle to both the period of forcing and the mean intensity of the vortex. For periodic heating in the upper troposphere, high–frequency forcing produces wave–like solutions, with azimuthal wind perturbations on the order of 0.2 m s$^{-1}$. Low–frequency forcing produces a local overturning circulation, with azimuthal wind perturbations of up to 1 m s$^{-1}$ observed in the upper–troposphere. For periodic heating in the lower troposphere, both high and low frequency forcing produce elliptic solutions in the region of heating, which resemble the Sawyer–Eliassen solution. Azimuthal wind perturbations vary by an order of magnitude from high to low frequencies, with a maximum values of 10 m s$^{-1}$ observed in the region of heating for a period of 72 h.

Comparison of these solutions to two simulations with modified diurnal periods in CM1 show a close correspondence of results. A diurnal period of 6 h demonstrates mostly wave–like behavior in the upper troposphere, with a weak lower tropospheric response. In contrast, a diurnal period of 72 h reveals a well-defined circulation in both the upper and lower troposphere, with absolute values of the radial wind anomalies of 3 m s$^{-1}$. Azimuthal wind perturbations are consistent with the diagnostic results, with magnitudes for the low–level forcing of the 72 h diurnal cycle an order of magnitude larger than the 6 h solutions.

These results demonstrate that the structure and magnitude of the TC diurnal cycle exhibits large variance, even for the same heating distributions. Short diurnal periods project onto inertia–buoyancy waves, which radiate energy away from the region of heating. In contrast, long diurnal periods exhibit slowly–varying overturning solutions, which produce azimuthal wind anomalies whose magnitude scales with the period of heating. This implies that the magnitude of the TC diurnal signal is dependent on the length of the heating phase, with longer periods producing a stronger wind response. Similarly, for a fixed period
of 24 h, results demonstrate that weaker initial vortices produce weaker azimuthal wind perturbations, with the magnitude of the diurnal signal scaling linearly with the intensity of the vortex. As the mean intensity increases, the inertial stability of the vortex increases, which widens the difference between \( \omega \) and \( I \) in the discriminant equation. This forces the boundary between elliptic and hyperbolic solutions further outward, and implies that less energy will radiate outwards as gravity waves.

These results imply that the structure and strength of the TC diurnal cycle is function of the inertial stability of the vortex. Since the period of the diurnal signal does not vary in nature, Eq. 4.1 demonstrates that as the inertial stability changes, the response to periodic heating will change, producing steadier or more wave-like solutions based on the relationship between \( \omega \) and \( I \). This work does not consider variations in the static stability or the buoyancy, which may also affect the strength and structure of the diurnal signal. However, given the close correspondence of the results of the diagnostic model to the CM1 response, we expect the influence to be minor.

This work does not consider the effect of storm asymmetries, which are present in real storms. Since the axisymmetric model is an azimuthal average of the heating, the magnitude of the signal may be overestimated, which would affect the strength of the induced periodic circulations. Further work is needed verify the sensitivity of the TC diurnal cycle in nature and explore the impact of these features.
Chapter 5

CONCLUSIONS

The goal of this dissertation is to diagnose the role of the diurnal cycle of radiation on axisymmetric hurricane structure. Using a long, statistically steady–state simulation that excludes all environmental influences, the variance associated with the TC diurnal signal is quantified, and the impact to the temperature, wind, and latent heating tendency fields is explored. Results demonstrate that the diurnal cycle accounts for 62% of the variance of the temperature field in the TC outflow layer, and 21% of the variance of the azimuthal wind in the lower troposphere, and is statistically significant at the 95% level. Composite anomalies computed at each hour of the day reveal that a cycle in radiative heating in the upper troposphere is indirectly linked to a cycle of latent heating tendency in the lower troposphere, which precedes a cycle in storm intensity by 6 h. Average magnitude of the composite azimuthal wind anomalies are 1 m s$^{-1}$, with maximum magnitudes at individual times of up to 10 m s$^{-1}$. Gravity waves are observed in the upper troposphere and the lower stratosphere, linked with temperature anomalies that arise in the TC outflow layer. Comparison of the radial–vertical wind vectors with the net radiative tendency and the latent heating tendency fields suggest that the TC diurnal cycle is a dynamic response to periodic heating, which drives the anomalous circulations in the wind and temperature fields.

These hypotheses are tested using a modified Sawyer–Eliassen approach for time–varying heating evaluated for a balanced vortex. Results demonstrate that the TC diurnal signal is a dynamic response to periodic heating, and is based on the location of the heat source. Upper–level periodic heating drives a local overturning response in the region of heating that manifests as gravity waves in the storm environment. Low–level periodic heating produces an overturning response throughout the lower–mid troposphere that resembles the Sawyer–
Eliassen solution. Comparison of the perturbation azimuthal wind fields from the diagnostic method to those of the CM1 simulation demonstrate a close correspondence of results, which suggests that periodic diurnal heating is driving the perturbation azimuthal wind response in CM1. These results are sensitive to the length of the diurnal period, as well as the vortex intensity, showing that the structure and strength of the TC diurnal cycle varies significantly even for the same heating distributions. High–frequency forcing projects onto inertia–buoyancy waves, which radiate energy away from the storm and produce a weak diurnal signal. Conversely, low–frequency forcing drives balanced solutions which exhibit stronger diurnal signals. Upward motion in the region of heating induces radial inflow at the base of the heat source, which drives higher values of angular momentum into the core of the storm, causing the vortex to intensify. In addition, for low–level periodic heating stronger vortices exhibit stronger diurnal signals, which suggest that the magnitude of the TC diurnal response also varies with the mean vortex intensity.

These results document a clear TC diurnal signal in a numerical framework that affects the entire vortex. The discriminant of the diagnostic solution reveals that the structure and magnitude of TC diurnal cycle depends on the relationship between the forcing frequency and the local inertial frequency; for regions where the local inertial frequency is large (e.g., near the core of the storm), and is larger than the forcing frequency, solutions are a balanced response to periodic heating that produce higher magnitude diurnal signals. For regions where the local inertial frequency is small, and is smaller than the forcing frequency, solutions manifest as inertia–buoyancy waves and produce weaker diurnal signals. Since the period of diurnal forcing in nature is fixed, these results suggest that the intensity of the storm, and particularly the inertial stability of the vortex, may impact the structure and strength of the TC diurnal cycle.

This work does not consider variations to the microphysics scheme, or the influence of storm asymmetries, which would impact the structure and magnitude of the heating fields. Future work should consider the impact of non-homogeneous heating distributions on the induced periodic circulations, either with three-dimensional simulations or in case studies.
of real storms. In addition, the influence of the storm environment on the structure and magnitude of the TC diurnal signal, such as variations in sea surface temperature, vertical wind shear, and changes in the profile of static stability, can also be evaluated; further sensitivity experiments quantifying the variance of the TC diurnal cycle with a more realistic storm environment is another possible avenue of future work.
BIBLIOGRAPHY


